they failed to notice Figs. 4 and 5, where such a comparison is presented. Figure 5 is particularly interesting; it shows that the analysis, despite its so-called drastic assumptions, correctly predicts the entrainment ratio as a function of molecular weight.

The statement in the second paragraph of the Comment that Emanuel compared "...compression ratios without specifying precisely what is to be held fixed" is puzzling. It may well be that the list at the end of Section IIC[1] of the four independent, dimensionless parameters was inadequately emphasized. It should be noted that one of these parameters is the ratio of the mass flow rate of driven to driver gas.

The choice of mass flow ratio vs molar flow ratio is rather arbitrary. What really counts for the applications mentioned in Ref. 1 is the overall size and weight of the ejector system. In this regard, the final remark in the Comment is incorrect. Considerable testing and system analysis at different companies, such as TRW and Rocketdyne, of various hot gas generators shows the desirability of a low molecular weight driver gas. An excellent example is hydrazine,[3] which can decompose into a low molecular weight gas, since one of the major products of decomposition is hydrogen.

References

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Comment on "Transonic Nozzle Flows of Gases with a Rate Process"

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A RECENT synoptic by Ishii[1] makes some concluding statements that require correction and suggest comment.

First, Mr. Ishii states that the solutions obtained in Refs. 2 and 3 "considered the case of Eq. (22)" or constant static pressure on the sonic line ($p_* = \text{const.}$). This is not correct. The transformed Euler equations, as represented by Eqs. (4) and (5) of Ref. 2 and as used in the development of both solutions of Refs. 2 and 3, clearly indicate the consideration given to the radial variation in static pressure. Possibly this misinterpretation stems from the sentence on page 213 of Ref. 3, which reads: "Since the first step is to determine the total pressure gradient across the flowfield, it is convenient to select a station in the combustion chamber where the flow is purely axial ($v = 0$) so that the radial gradient of static pressure will be zero." Note that this merely suggests a procedure for determining the gradient in terms of other known variations in the gas properties; it does not mean a constant static pressure in the transonic region.

Second, Mr. Ishii states, when referring to solutions of Refs. 2 and 3, that "It is obvious that his result is not general in this respect because the condition of Eq. (22) is only one of the necessary conditions for existence of the Hall type of perturbation solution." We contend that the condition of constant static pressure on the sonic line as expressed in Eq. (22) is not a necessary condition, as evidenced by the Hall type solution of Ref. 3 where p_* is variable. Furthermore, it is not apparent what other conditions he refers to when he states that Eq. (22) is "only one of the necessary conditions," since he has already stated, and we agree based upon our results in Refs. 2 and 3, that a stipulation of a constant or variable γ is not necessary.

We are sure Mr. Ishii had valid reasons for expressing the nonuniformities in terms of the flow static properties. However, it is our opinion that expressing them in terms of the total rather than the static flow properties results in the clearer insight into the solution. By doing this in Ref. 3, it was determined that all nonuniformities in the flowfield reduce to a variation in γ only. Since solutions are possible for either a constant or variable γ, one concludes that there really are "no necessary conditions" for the existence of a so-called Hall type of solution other than the basic requirement that the wall contour perturbation parameter be small relative to unity.

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Reply by Author to S. Boraas

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EVEN now, the author believes that the concluding remarks in Ref. 1 are completely correct and valid. Some of the results in Ref. 2 have been based upon erroneous equations. For example, Eq. (5) of Ref. 2 is not correct. The correct one is

$$\frac{\sin\theta \dfrac{\partial V^*}{\partial \xi}}{\left[\left(\dfrac{\gamma+1}{2}\right)\epsilon\right]^{1/2}} + V^*\cos\theta\frac{\partial \phi}{\partial \xi} = \frac{2}{\gamma+1}\,\frac{2\pi r_{tid}{}^2}{\dot{m}}\,\zeta\frac{\gamma p_t}{a^*}$$

$$\left[1-\left(\frac{\gamma-1}{\gamma+1}\right)V^{*2}\right]^{\frac{1}{\gamma-1}}\left\{\left[V^*\frac{\partial V^*}{\partial \eta} + \frac{V^{*2}}{(\gamma^2-1)}\frac{\partial \gamma}{\partial \eta}\right]\right.$$

$$+\frac{(\gamma+1)}{2\gamma(\gamma-1)^2}\left[1-\left(\frac{\gamma-1}{\gamma+1}\right)V^{*2}\right]\log_e\left[1-\left(\frac{\gamma-1}{\gamma+1}\right)V^{*2}\right]\frac{\partial \gamma}{\partial \eta}$$

$$\left.-\frac{\gamma+1}{2}\,\frac{1}{\gamma p_t}\left[1-\left(\frac{\gamma-1}{\gamma+1}\right)V^{*2}\right]\frac{\partial p_t}{\partial \eta}\right\} \tag{1}$$

Therefore, the function Γ_0 in Eq. (14) of Ref. 2, which is defined in the Appendix of Ref. 2, must be replaced by

$$\Gamma_0 = \frac{\gamma}{\gamma^2-1} + \frac{1}{(\gamma-1)^2}\log_e\left(\frac{2}{\gamma+1}\right) \tag{2}$$

It is quite easy to see that the corrected Eq. (14) of Ref. 2 is equivalent to the relation $d(\log_e p^*)/d\beta = 0$ in Ref. 1. Con-

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sidering the classical ideal gas, we have

$$p^* = p_t \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

where p^* is the static pressure at the sonic point on each streamline. From this equation it is found that

$$\frac{d}{d\eta}\log_e p^* = \frac{d\zeta_0}{d\eta}\frac{d}{d\zeta_0}\log_e p^* = \frac{d\zeta_0}{d\eta}\left\{\frac{d}{d\zeta_0}\log_e p_t - \Gamma_0 \frac{d\gamma}{d\zeta_0}\right\} \tag{4}$$

since ζ_0 defined by Eq. (11) of Ref. 2 is obviously a function of η only. Using the corrected Eq. (14) of Ref. 2, we can easily conclude that

$$\frac{d}{d\eta}\log_e p^* = 0 \quad \left(\frac{d}{d\beta}\log_e p^* = 0 \text{ in Ref. 1}\right) \tag{5}$$

From this discussion it is obvious that the corrected Eq. (14) of Ref. 2 indicates only the fact that the static pressure at the sonic point must be constant across the streamlines.

Boraas has misinterpreted the relation $d(\log_e p^*)/d\beta = 0$ of Ref. 1. The quantity p^* of Ref. 1 is equal to that defined by Eq. (3), and is clearly a function of η (or β in Ref. 1) only. Then the condition of constant p^* does not mean constant static pressure along ξ = const. or $\partial p/\partial\eta = 0$ in the transonic portion.

The condition of Eq. (5), or more exactly $d(\log_e p^*)/d\eta = O(\epsilon)$ ($d(\log_e p^*)/d\beta = O(\epsilon)$ of Ref. 1), is necessary for the existence of Hall's type of transonic solution. This condition is warranted by the fact that the sonic line location is completely contained in the throat region or in the region $x = O(L)$, where L is the reference nozzle length defined both in Refs. 1 and 2. If this condition is not satisfied for some flow, the sonic line of such a flow cannot be completely contained in the geometric-throat region and then the problem cannot be solved in a closed form by applying Hall's technique. The necessary conditions for the existence of Hall's type of transonic solution for real gases will be discussed in detail elsewhere by the present author.

The conclusion in Ref. 2 (of reducibility of all nonuniformities into a variation in γ only) might perhaps be in error, since this is true only for the flows where the condition of Eq. (5) is satisfied.

Finally, the author would like to emphasize that, in spite of such errors, some of the results of Ref. 2 remain valid and valuable.

References

[1]Ishii, R., "Transonic Nozzle Flows of Gases with a Rate Process," *AIAA Journal*, Vol. 15, March 1977, pp. 299-300.

[2]Boraas, S., "Transonic Nozzle Flow with Nonuniform Gas Properties," *AIAA Journal*, Vol. 11, Feb. 1973, pp. 210-215.

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Ionizational Nonequilibrium Heating During Outer Planetary Entries

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THE second author's name, as it appears on the top of pages 1326 and 1328, should read "T. -J. Kuo" instead of "T. -N. Kuo."

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Turbulent Mixing Coefficients for Compressible Coaxial Submerged and Coflowing Jets

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[AIAA J., **14**, 1699-1705 (1976)]

THE coefficients shown in Eqs. (4) and (5) of the subject paper were high by an order of magnitude. The correlative equations should read as follows:

$$K_v - K_{vo} = 0.066\, M_5^{0.31} (\rho_1/\rho_5)^{-0.80} (u_\infty / |u_1 - u_\infty|)^{0.48} \tag{4}$$

$$K_c - K_{co} = 0.066\, M_5^{0.32} (\rho_1/\rho_5)^{-1.18} (u_\infty / |u_1 - u_\infty|)^{0.385} \tag{5}$$

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Index categories: Jets, Wakes and Viscid-Inviscid Flow Interactions; Multiphase Flows.

Notice: SI Units

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George W. Sutton
Editor-in-Chief